hep-th/0010007

Gauge Symmetry Breaking and Generalized Monopole in Non-BPS D-Brane Action

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Abstract

Based on the DBI action for the four coincident non-BPS D9-branes in the type IIA string theory we demonstrate that the gauge symmetry breaking through the tachyon condensation into the generalized monopole of codimension five produces a pair of two coincident BPS D4-branes. The nontrivial gauge field configuration is studied and shown to yield the non-zero generalized magnetic charge. We discuss how this explicit demonstration is related with the higher K-theory group.

October, 2000

1 Introduction

In the study of non-perturbative behaviors of the supersymmetric string and gauge field theories the D-branes have offered various important aspects. The spectrum in some vacua of the string theory includes not only the BPS D-branes that are themselves solitonic stable states, but also the unstable non-BPS D-branes that can be made stable by modding out with some finite group [1, 2, 3]. Besides the unstable non-BPS D-branes there is an unstable object that consists of an equal number of BPS Dp-branes and BPS Dp-antibranes (D$\bar{p}$-branes), whose instability is due to the presence of a tachyon in the spectrum of the $p - \bar{p}$ open strings. In the type IIB string theory the stable D-brane states of lower dimensions appear as bound states of the Dp-brane D$\bar{p}$-brane system via the tachyon condensation and the D-brane charges are identified with the elements of $\tilde{\mathrm{K}}(X)$, the reduced K-theory group of the spacetime manifold X [4]. In this approach from a D9-D$\bar{9}$ pair and two D9-D$\bar{9}$ pairs via the tachyon condensations a BPS D7-brane and a BPS D5-brane are built respectively where the nontrivial gauge field configurations are given by the two-dimensional vortex and the four-dimensional instanton, which are associated with the homotopy groups and the reduced K-theory groups in such ways as $\Pi_1(U(1)) = \tilde{\mathrm{K}}(S^2) = Z$ and $\Pi_3(U(2)) = \tilde{\mathrm{K}}(S^4) = Z$. For the type IIA string theory a non-BPS D9-brane itself is so unstable that a number of unstable D9-branes dcay to produce the BPS D-branes by the tachyon condensations, which are classified by the higher K-theory group $\mathrm{K}^{-1}(X)$ [5]. For instance a system of two D9-branes and a system of the four D9-branes decay to yield a D6-brane and a D4-brane respectively through the 't Hooft-Polyakov three-dimensional monopole and a generalized five-dimensional monopole which are classified by $\Pi_2(U(2)/U(1) \times U(1)) = \mathrm{K}^{-1}(S^3) = Z$ and $\Pi_4(U(4)/U(2) \times U(2)) = \mathrm{K}^{-1}(S^5) = Z$. On the other hand the unstable D9-branes can produce the D8-D$\bar{8}$ pairs by the tachyon condensation into a kink not accompanied with the nontrivial gauge configuration.

The non-BPS Wess-Zumino couplings of type IIA non-BPS D-branes to the Ramond-Ramond (R-R) potentials have been presented and shown to produce the BPS Wess-Zumino action through the tachyon condensation [6]. For the system of D-branes and D-antibranes of the type II string theories the generalization of the Wess-Zumino action has been also performed [7]. Moreover the effective DBI actions for non-BPS D-branes in the type II string theories have been proposed by Sen [8] where the interactions between the tachyon and other light fields are restricted by the supersymmetry and the general covariance and the requirement of possible tachyon condensation. As an attempt to generalize Sen's proposal an effective non-BPS D-brane action has been presented in Ref. [9]. Based on this effective action in the type IIA or IIB string theory a BPS Dp-brane is shown to be produced from a non-BPS D$(p+1)$-brane via the tachyon condensation in a form of kink. Repeating this step all D-branes are constructed from a number of D9-branes or a system of D9-branes and D$\bar{9}$-branes. This step by step construction uses a kink solution where the gauge fields are trivial. On the other hand in the K-theory group view-point for the BPS D-brane charges the nontrivial gauge field configurations play important roles. The other attempt to generalize Sen's proposal has been performed by requiring the T-duality [10] to yield the different type of the non-BPS D-brane action, whose expression was suggested by calculations of S-matrix elements [11]. There is a proposal of an interpolating DBI action for a single non-BPS

D-brane which reduces to the previous two types of actions in the two particular limits [12].

Based on the effective DBI action for the non-BPS D9-branes in the type IIA string theory presented in Refs. [10, 11] we will investigate the tachyon condensation in a form of the nontrivial gauge field configuration. Specially considering the Higgs mechanism in a genralized five-dimensional monopole configuration we will construct the world-volume gauge theories for BPS D4-branes by one step procedure. The nontrivial gauge fields will be estimated and from them a non-zero generalized magnetic charge will be evaluated. The explicit construction of BPS D4-branes will be argued in comparison with the general approach based on the higher K-theory group.

2 One step construction

We start to write down the DBI action for the massless bosonic excitations of the N coincident non-BPS D9-branes in the type IIA string theory

$$S = -C_9 \int d^{10}\sigma \mathrm{Tr}(g(T)\sqrt{-\det(G_{\mu\nu} + B_{\mu\nu} + F_{\mu\nu} + D_\mu T D_\nu T)}), \tag{1}$$

where $G_{\mu\nu}, B_{\mu\nu}, F_{\mu\nu}$ and T are the metric, Neveu-Schwarz antisymmetric tensor, $U(N)$ gauge field strength and tachyon field respectively. The massless fermionic excitations can be included by requiring the spacetime supersymmetry invariance with no κ-symmetry. The gauge trace is taken as a symmetrized trace [13]. This type of abelian DBI action for a single non-BPS D-brane was proposed from the viewpoint of T-duality [10]. Here we have taken the trace operation in the same way as was made in Ref. [11], which is slightly different from the trace operation in the other type of DBI action for non-BPS D-branes in Ref. [9] where the trace is separately taken in the potential part and the sruare root part. The tachyon field T has a tendency to role down to a certain value T_0 at the minimum of the potential $g(T)$. We express the potential as $g(T) = \mathrm{I} + V(T)$ where I is the 4×4 unit matrix and $V(T)$ denotes universal tachyon potential [14]. The potential $g(T)$ is conjectured to vanish at $T = T_0$. When we restrict ourselves to tachyon part of the DBI action (1) by putting all the fields zero except the tachyon field, the negative energy density of the condensed tachyon is conjectured to cancel the positive energy density of the N D9-branes asymptotically, which is expressed as $C_9 \mathrm{Tr} V(T_0) + N C_9 = 0$ where C_9 is the tension of a single non-BPS D9-brane.

Now we consider a particular system of four coincident non-BPS D9-branes with flat metric and no $B_{\mu\nu}$ background. In the following we will work in the static gauge. The expansion of the square root in (1) yields

$$S = -C_9 \int d^{10}\sigma \mathrm{Tr}(\mathrm{I} + V(T))(\mathrm{I} + \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} D_\mu T D^\mu T), \tag{2}$$

where $D_\mu T = \partial_\mu T + i[A_\mu, T], F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$ and the $U(4)$ gauge field A_μ as well as the tachyon field T are in the adjoint of $U(4)$. It is natural to suppose that $V(T)$ is even and so takes the form

$$V(T) = -\mu^2 T^2 + \lambda T^4. \tag{3}$$

The parameters, μ and λ will be related so that the potential $g(T)$ vanishes at $T = T_0$.

The higher K-theory group indicates that in an unstable system of the four D9-branes the gauge symmetry breaking occures from $U(4)$ to $U(2)\times U(2)$ through the tachyon condensation, where the tachyon field takes values in the vacuum manifold $U(4)/U(2)\times U(2)$ far from the core of the topological magnetic defect of codimension 5 [5]. The generators of $U(4)$ are divided into those of the unbroken gauge symmetry group $U(2)\times U(2)$ and those of the coset, while they are defined by the $U(1)$ generator $\lambda_0 = \mathrm{I}$ and the $SU(4)$ traceless generators $\lambda_a (a = 1,\cdots,15)$. There are convenient bases of $su(4)$ Lie algebra

$$
\begin{aligned}
(E_{ij})^{kl} &= \delta_i^k\delta_j^l - \delta_i^l\delta_j^k, \\
(F_{ij})^{kl} &= i(\delta_i^k\delta_j^l + \delta_i^l\delta_j^k), \text{ for } i \neq j, \\
(F_{ii})^{kl} &= i(\delta_i^k\delta_i^l - \delta_4^l\delta_4^k)
\end{aligned}
\tag{4}
$$

with $i = 1,\cdots,4$, which are related with λ_a in three groups as $\{i\lambda_1 = F_{12}, i\lambda_2 = E_{12}, i\lambda_3 = F_{11} - F_{22}\}, \{i\lambda_4 = F_{13}, i\lambda_5 = E_{13}, i\lambda_6 = F_{23}, i\lambda_7 = E_{23}, i\lambda_8 = (F_{11} + F_{22} - 2F_{33})/\sqrt{3}, i\lambda_9 = F_{14}, i\lambda_{10} = E_{14}, i\lambda_{11} = F_{24}, i\lambda_{12} = E_{24}\}, \{i\lambda_{13} = F_{34}, i\lambda_{14} = E_{34}, i\lambda_{15} = (F_{11}+F_{22}+F_{33})/\sqrt{6}\}$. In view of these expressions the unbroken gauge symmetry has one $U(2)$ generators $\bar{t}^a$ consisting of F_{12}, E_{12} and $F_{11} - F_{22}$ with $(\mathrm{I} - i(F_{11} + F_{22} - F_{33}))/2$ and the other $U(2)$ ones t'^a consisting of F_{34}, E_{34} and F_{33} with $(\mathrm{I} + i(F_{11} + F_{22} - F_{33}))/2$. The remaining eight bases such as $E_{13}, E_{14}, E_{23}, E_{24}$ and $F_{13}, F_{14}, F_{23}, F_{24}$ yield the generators $\tilde{t}^\alpha$ of the coset. Here we express the $U(4)$ gauge field as $A_\mu = \sum_{a=1}^4 \bar{A}_\mu^a\bar{t}^a + \sum_{a=1}^4 A'^a_\mu t'^a + \sum_{\alpha=1}^8 \tilde{A}_\mu^\alpha\tilde{t}^\alpha$.

Taking the low energy limits that all derivatives and all field strength of massless fields are put small we further expand the effective action (2) as

$$
S = -C_9\int d^{10}\sigma \mathrm{Tr}(\mathrm{I} + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu T D^\mu T + V(T)),
\tag{5}
$$

where the non-leading terms $V(T)(F_{\mu\nu}^2/4 + (D_\mu T)^2/2)$ have been suppressed. From the leading terms in (5) the equations of motion for the tachyon field and the gauge field are given by

$$
\begin{aligned}
D_{\hat{\mu}}D^{\hat{\mu}}T + D_iD^iT &= -2\mu^2T + 4\lambda T^3, &(6)\\
\partial^{\hat{\mu}}F_{\hat{\mu}\hat{\nu}} + i[A^{\hat{\mu}}, F_{\hat{\mu}\hat{\nu}}] + \partial^iF_{i\hat{\nu}} + i[A^i, F_{i\hat{\nu}}] &= i[T, D_{\hat{\nu}}T], &(7)\\
\partial^{\hat{\mu}}F_{\hat{\mu}j} + i[A^{\hat{\mu}}, F_{\hat{\mu}j}] + \partial^iF_{ij} + i[A^i, F_{ij}] &= i[T, D_jT], &(8)
\end{aligned}
$$

where we have split the world-volume coordinates σ^μ into $(x^{\hat{\mu}}, x^i)$, $\hat{\mu} = 0,\cdots,4$ and $i = 5,\cdots,10$. Now we consider the tachyon condensation into the topological defect in codimension 5 so that we can assume that the tachyon condensate is a function of the five transverse coordinates $x^i, i = 5,\cdots,10$. Therefore from (7) we have a trivial classical solution $A^c_{\hat{\mu}} = 0$ and a condition that A^c_i should be independent of $x_{\hat{\mu}}$. Then the first term of the left-handed side (LHS) in (6) and the first two terms of the LHS in (8) vanish so that the remaining terms in (6) and (8) combine to yield the nontrivial generalized monopole solution $(A^c_i(x_j), T^c(x_j))$. Far from the core the tachyon takes the form $T^c_0(x_j) = c_0U$ with $U^2 = \mathrm{I}$ and a constant c_0, whose behavior will be discussed later. We choose a parametrization $4\lambda = \mu^4$ so that the potential $\mathrm{Tr}(\mathrm{I} + V(T))$ in (5) vanishes at the minimum of the potential and is expressed as $\mathrm{Tr}\lambda(T^2 - \mu^2\mathrm{I}/2\lambda)^2$. Therefore c_0^2 is fixed by $2/\mu^2$. We assume that in the whole region the

tachyon solution is expressed as $T^c = c(r)U$ with $r = \sqrt{x_i^2}$, the radius in the five transverse dimensions, where a real function $c(r)$ approaches $\sqrt{2}/\mu$ for $r \to \infty$ and becomes zero near $r = 0$ [4, 5]. Though the function $c(r)$ may be numerically determined by solving (6) and (8), its explicit form is not so important for the following arguments.

In order to consider the behaviors of fluctuations around the classical configuration, we put

$$\begin{aligned} \bar{A}_{\hat{\mu}} &= \bar{A}^c_{\hat{\mu}} + \bar{\boldsymbol{A}}_{\hat{\mu}},\ A'_{\hat{\mu}} = A'^c_{\hat{\mu}} + \boldsymbol{A}'_{\hat{\mu}},\ \tilde{A}_{\hat{\mu}} = \tilde{A}^c_{\hat{\mu}} + \tilde{\boldsymbol{A}}_{\hat{\mu}}, \\ \bar{A}_i &= \bar{A}^c_i(x_j) + \bar{X}_i,\ A'_i = A'^c_i(x_j) + X'_i,\ \tilde{A}_i = \tilde{A}^c_i(x_j) + \tilde{\boldsymbol{A}}_i \end{aligned} \tag{9}$$

with $\bar{A}^c_{\hat{\mu}} = A'^c_{\hat{\mu}} = \tilde{A}^c_{\hat{\mu}} = 0$ and $T = T^c(x_j) + \eta$. Through the Higgs mechanism the substitutions of the expansions (9) into (5) produce the spectrum consisting of the massive Higgs boson η, the massive bosons $\tilde{\boldsymbol{A}}_{\hat{\mu}}$ as well as $\tilde{\boldsymbol{A}}_i$ associated with the broken gauge symmetry, and the massless bosons $\bar{\boldsymbol{A}}_{\hat{\mu}}, \boldsymbol{A}'_{\hat{\mu}}$ as well as $\bar{X}_i, X'_i$ associated with the unbroken gauge symmetry $U(2) \times U(2)$. Even though the spontaneous symmetry breaking of gauge group $U(4)$ occurs in the i-components, this must hold for the other $\hat{\mu}$-components as well, since the gauge symmetry breaking is independent of the component of particular gauge fields. Therefore the gauge boson $\tilde{\boldsymbol{A}}_{\hat{\mu}}$ becomes massive in the same way as $\tilde{\boldsymbol{A}}_i$. Here we assume that the massless fields $\bar{\boldsymbol{A}}_{\hat{\mu}}, \boldsymbol{A}'_{\hat{\mu}}, \bar{X}_i, X'_i$ have no x_i dependence. The kinetic energy part of the gauge field $\mathrm{Tr}F^2_{\mu\nu}$ is decomposed into $\mathrm{Tr}(F^2_{\hat{\mu}\hat{\nu}} + 2F^2_{i\hat{\mu}} + F^2_{ij})$. In $\mathrm{Tr}F^2_{ij}$ the expansions (9) yield the classical part and the quadratic fluctuation part which includes the following term

$$\begin{aligned} \mathrm{Tr}\boldsymbol{F}^2_{ij} &= \mathrm{Tr}(\ \bar{\boldsymbol{F}}^2_{ij} + \boldsymbol{F}'^2_{ij} + \boldsymbol{W}^2_{ij} + 2ig([\tilde{\boldsymbol{A}}_i, \tilde{\boldsymbol{A}}_j](\bar{\boldsymbol{F}}^{ij} + \boldsymbol{F}'^{ij}) + \boldsymbol{W}_{ij}\boldsymbol{A}^{ij}) \\ &- g^2([\tilde{\boldsymbol{A}}_i, \tilde{\boldsymbol{A}}_j]^2 + \boldsymbol{A}^2_{ij})\ ), \end{aligned} \tag{10}$$

where $\bar{\boldsymbol{F}}_{ij} = \partial_i \bar{X}_j - \partial_j \bar{X}_i + i[\bar{X}_i, \bar{X}_j], \boldsymbol{F}'_{ij} = \partial_i X'_j - \partial_j X'_i + i[X'_i, X'_j], \boldsymbol{W}_{ij} = \partial_i \tilde{\boldsymbol{A}}_j - \partial_j \tilde{\boldsymbol{A}}_i, \boldsymbol{A}_{ij} = [\bar{X}_i + X'_i, \tilde{\boldsymbol{A}}_j] + [\tilde{\boldsymbol{A}}_i, \bar{X}_j + X'_j]$ and the $U(4)$ gauge coupling constant g has been restored only here for convenience. The first two terms in the RHS of (10) indicating the massless excitations become $-\mathrm{Tr}([\bar{X}_i, \bar{X}_j]^2 + [X'_i, X'_j]^2)$ owing to $\partial_i \bar{X}_j = \partial_i X'_j = 0$. Similarly the massless part of quadratic fluctuations in $\mathrm{Tr}F^2_{i\hat{\mu}}$ is extracted as $\mathrm{Tr}((\bar{D}_{\hat{\mu}}\bar{X}_i)^2 + (D'_{\hat{\mu}}X'_i)^2)$ where $\bar{D}_{\hat{\mu}}\bar{X}_i = \partial_{\hat{\mu}}\bar{X}_i + i[\bar{\boldsymbol{A}}_{\hat{\mu}}, \bar{X}_i]$ and $D'_{\hat{\mu}}X'_i = \partial_{\hat{\mu}}X'_i + i[\boldsymbol{A}'_{\hat{\mu}}, X'_i]$. The $\mathrm{Tr}F^2_{\hat{\mu}\hat{\nu}} = \mathrm{Tr}\boldsymbol{F}^2_{\hat{\mu}\hat{\nu}}$ itself is also expressed in the same way as (10). The Higgs mechanism induces masses for $\tilde{\boldsymbol{A}}_{\hat{\mu}}, \tilde{\boldsymbol{A}}_i$ and η, thereby removing them from the low-energy spectrum. Recombining the leading terms (5) which determine the classical configuration, with the non-leading terms we obtain the effective low-energy action for the massless fluctuating fields

$$S = -C_9 \int d^{10}\sigma \mathrm{Tr}(\mathrm{I} + V(T^c))\frac{1}{4}(\bar{\boldsymbol{F}}_{\mu\nu}\bar{\boldsymbol{F}}^{\mu\nu} + \boldsymbol{F}'_{\mu\nu}\boldsymbol{F}'^{\mu\nu}) \tag{11}$$

with $\bar{\boldsymbol{A}}_\mu = (\bar{\boldsymbol{A}}_{\hat{\mu}}, \bar{X}_i)$ and $\boldsymbol{A}'_\mu = (\boldsymbol{A}'_{\hat{\mu}}, X'_i)$. Owing to the factorization into the $x^{\hat{\mu}}$ and x^i dependent parts we can integrate over the transverse coordinates x^i, where the trace goes through the tachyon potential part to operate on the gauge kinetic part. Therefore we derive

$$\begin{aligned} S &= -C_9 a \int d^5\sigma \mathrm{Tr}\frac{1}{4}(\bar{\boldsymbol{F}}_{\hat{\mu}\hat{\nu}}\bar{\boldsymbol{F}}^{\hat{\mu}\hat{\nu}} + \boldsymbol{F}'_{\hat{\mu}\hat{\nu}}\boldsymbol{F}'^{\hat{\mu}\hat{\nu}} \\ &+ 2\bar{D}_{\hat{\mu}}\bar{X}_i\bar{D}^{\hat{\mu}}\bar{X}^i + 2D'_{\hat{\mu}}X'_iD'^{\hat{\mu}}X'^i - [\bar{X}_i, \bar{X}_j][\bar{X}^i, \bar{X}^j] - [X'_i, X'_j][X'^i, X'^j]), \end{aligned} \tag{12}$$

where $a = \int d^5x\lambda(c^2(r) - \mu^2/2\lambda)^2$ and there remains a gauge symmetry $U(2) \times U(2)$.

The obtained action is considered to represent a world-volume gauge theory describing a system of two coincident BPS D4-branes and the other two coincident BPS D4-branes. The transverse fluctuations of these D4-branes are expressed by $\bar{X}_i$ and X'_i. The factor C_9a can be regarded as the tension of the BPS D4-brane. In this way a pair of BPS D4-branes emerge as the quantum fluctuations around the generalized monopole solution. The last two terms in (12) are characteristic of the tachyon condensation into the generalized monopole in this one step construction, compared with the kink case for the other processes where such terms are absent [9].

3 Magnetic charge of the generalized monopole

The generalized monopole configuration of codimension 5 supported by the unstable four D9-branes is classified by the nontrivial element in the homotopy group of the vacuum manifold $\Pi_4(U(4)/U(2) \times U(2)) = Z$, which is related to the higher K-theory group of spacetime [5]. For this codimension $n = 5$ defect, the tachyon field T is so specified by a generator of $\Pi_4(U(4)/U(2) \times U(2))$ that T maps the sphere $S^{n-1} = S^4$ at infinity in the five transverse dimensions to the vacuum manifold $U(4)/U(2) \times U(2)$. The world-volume of four D9-branes supports a $U(4)$ Chan-Paton bundle, which is identified with a spinor bundle $\mathcal{S}$ of the group $SO(5)$ of rotations in the transverse dimensions. Then the tachyon condensate for the generalized monopole configuration is given by

$$T = c(r)\Gamma_i \frac{x^i}{r}, \tag{13}$$

which satisfies $T^2 = c^2(r)\mathrm{I}$ and corresponds to the previously defined tachyon solution $T^c = c(r)U$ with the convergence factor $c(r)$. Here we change the numbering for x_i into $i = 1, \cdots, 5$. In this stable defect Γ_i are the Γ-matrices of the group $SO(5)$ described by 4×4 matrices, which map from the four-dimensional spinor bundle $\mathcal{S}$ back to $\mathcal{S}$. For the Γ-matrices obeying the Clifford algebra $\{\Gamma_i, \Gamma_i\} = 2\delta_{ij}$ with $i = 1, \cdots, 5$ we take a convenient representation,

$$\begin{aligned} \Gamma_{\hat{i}} &= \sigma_{\hat{i}} \otimes \sigma_1 = \begin{pmatrix} 0 & \sigma_{\hat{i}} \\ \sigma_{\hat{i}} & 0 \end{pmatrix}, \text{ for } \hat{i} = 1, 2, 3 \\ \Gamma_4 &= -\mathrm{I}_2 \otimes \sigma_2 = \begin{pmatrix} 0 & i\mathrm{I}_2 \\ -i\mathrm{I}_2 & 0 \end{pmatrix}, \ \Gamma_5 = \mathrm{I}_2 \otimes \sigma_3 = \begin{pmatrix} \mathrm{I}_2 & 0 \\ 0 & -\mathrm{I}_2 \end{pmatrix}, \end{aligned} \tag{14}$$

where $\sigma_{\hat{i}}$ are the 2×2 Pauli matrices and I_2 is the 2×2 unit matrix. As in the analysis of the 't Hooft-Polyakov monopole and the vortex, we must require a finite-energy configuration for the generalized monopole that the covariant gradient of the order parameter, that is, the tachyon field falls off sufficiently rapidly at large distances

$$\partial_i T = -i[A_i, T]. \tag{15}$$

In (15) taking account of the asymptotic form of (13)

$$T = c_0\Gamma_i \frac{x^i}{r} = \frac{c_0}{r} \begin{pmatrix} x^5\mathrm{I}_2 & x^{\hat{i}}\sigma_{\hat{i}} + ix^4\mathrm{I}_2 \\ x^{\hat{i}}\sigma_{\hat{i}} - ix^4\mathrm{I}_2 & -x^5\mathrm{I}_2 \end{pmatrix} \tag{16}$$

we obtain a nontrivial $U(4)$ gauge field

$$A_i = \frac{i}{4r^2}[\Gamma_i, \Gamma_j]x^j \tag{17}$$

with $i = 1, \cdots, 5$, whose expression is suggested by the 't Hooft-Polyakov monopole solution of winding number one [15]. When we take the viewpoint of the $U(4)$ Chan-Paton bundle, the asymptotic solution (16) for tachyon shows that T/c_0 is the 4×4 matrix that is simultaneously hermitian and unitary, which is an element of $U(4)$ group as well as Lie algebra. This intersection of $U(4)$ with its Lie algebra $u(4)$ is identified with the Grassmannian manifold $U(4)/U(2)\times U(2)$ [16]. We will examine whether the asymptotic solutions (16), (17) satisfy the Eq. (15). Substituting the solution (17) into (15) and using

$$\begin{aligned}
[\Gamma_{\hat{i}}, \Gamma_{\hat{j}}] &= 2i\epsilon_{\hat{i}\hat{j}\hat{k}} \begin{pmatrix} \sigma^{\hat{k}} & 0 \\ 0 & \sigma^{\hat{k}} \end{pmatrix} \equiv 2i\epsilon_{\hat{i}\hat{j}\hat{k}} P^{\hat{k}}, \\
[\Gamma_{\hat{i}}, \Gamma_4] &= 2i \begin{pmatrix} -\sigma_{\hat{i}} & 0 \\ 0 & \sigma_{\hat{i}} \end{pmatrix} \equiv 2iQ_{\hat{i}},\ [\Gamma_{\hat{i}}, \Gamma_5] = 2i \begin{pmatrix} 0 & -\sigma_{\hat{i}} \\ \sigma_{\hat{i}} & 0 \end{pmatrix} \equiv 2R_{\hat{i}}
\end{aligned} \tag{18}$$

we observe that the right-handed side (RHS) of (15) for $i = \hat{i}$ becomes

$$\frac{c_0}{r^3}(-\epsilon_{\hat{i}\hat{j}\hat{l}}x^{\hat{j}}x_{\hat{k}}\epsilon^{\hat{l}\hat{k}\hat{m}}\Gamma_{\hat{m}} + \sum_{k=4}^{5} x^k(x_k\Gamma_{\hat{i}} - x_{\hat{i}}\Gamma_k)), \tag{19}$$

where we have used the following commutation relations too

$$\begin{aligned}
[P_{\hat{l}}, \Gamma_{\hat{k}}] &= 2i\epsilon_{\hat{l}\hat{k}\hat{m}}\Gamma^{\hat{m}},\ [P_{\hat{l}}, \Gamma_4] = [P_{\hat{l}}, \Gamma_5] = 0, \\
[Q_{\hat{i}}, \Gamma_{\hat{k}}] &= 2i\delta_{\hat{i}\hat{k}}\Gamma_4,\ [Q_{\hat{i}}, \Gamma_4] = -2i\Gamma_{\hat{i}},\ [Q_{\hat{i}}, \Gamma_5] = 0, \\
[R_{\hat{i}}, \Gamma_{\hat{k}}] &= -2\delta_{\hat{i}\hat{k}}\Gamma_5,\ [R_{\hat{i}}, \Gamma_4] = 0,\ [R_{\hat{i}}, \Gamma_5] = 2\Gamma_{\hat{i}}.
\end{aligned} \tag{20}$$

The expression (19) agrees with the LHS of (15)

$$\partial_{\hat{i}}T = c_0(\frac{1}{r}\Gamma_{\hat{i}} - \frac{x_{\hat{i}}}{r^3}(\Gamma_{\hat{l}}x^{\hat{l}} + \sum_{k=4}^{5}\Gamma_k x^k)) \tag{21}$$

with $r^2 = x_{\hat{i}}^2 + x_4^2 + x_5^2$. We further use

$$\begin{aligned}
[\Gamma_4, \Gamma_5] &= -2i \begin{pmatrix} 0 & \mathrm{I}_2 \\ \mathrm{I}_2 & 0 \end{pmatrix} \equiv -2iS,\ [S, \Gamma_{\hat{k}}] = 0, \\
[S, \Gamma_4] &= -2i\Gamma_5,\ [S, \Gamma_5] = 2i\Gamma_4
\end{aligned} \tag{22}$$

to express the RHS of (15) for $i = 4$ as $c_0(x^{\hat{j}}(x_{\hat{j}}\Gamma_4 - x_4\Gamma_{\hat{j}}) + x^5(x_5\Gamma_4 - x_4\Gamma_5))/r^3$ which matches the LHS of (15) $\partial_4 T = c_0(\Gamma_4/r - x_4\Gamma_j x^j/r^3)$. For $i = 5$ the RHS is similarly described as $c_0(x^{\hat{j}}(x_{\hat{j}}\Gamma_5 - x_5\Gamma_{\hat{j}}) + x^4(x_4\Gamma_5 - x_5\Gamma_4))/r^3$ which also equals to the LHS $\partial_5 T$.

Here we extend to the generalized monopole configuration of the higher codimension 7 supported by the unstable eight D9-branes, which is also classified by $\Pi_6(U(8)/U(4)\times$

$U(4)) = Z$. We define the Γ-matrices of $SO(7)$ whose spinor bundle is identified with a $U(8)$ Chan-Paton bundle, as the 8×8 matrices, $\mathbf{\Gamma}_{\hat{i}} = \sigma_{\hat{i}} \otimes \sigma_1 \otimes \sigma_1, \mathbf{\Gamma}_4 = -\mathrm{I}_2 \otimes \sigma_2 \otimes \sigma_1, \mathbf{\Gamma}_5 = \mathrm{I}_2 \otimes \sigma_3 \otimes \sigma_1, \mathbf{\Gamma}_6 = -\mathrm{I}_2 \otimes \mathrm{I}_2 \otimes \sigma_2$ and $\mathbf{\Gamma}_7 = \mathrm{I}_2 \otimes \mathrm{I}_2 \otimes \sigma_3$ which satisfy the Clifford algebra manifestly. This definition gives a natural extension of (14)

$$\begin{aligned} \mathbf{\Gamma}_k &= \begin{pmatrix} 0 & \Gamma_k \\ \Gamma_k & 0 \end{pmatrix}, \text{ for } k = 1, \cdots, 5 \\ \mathbf{\Gamma}_6 &= \begin{pmatrix} 0 & i\mathrm{I} \\ -i\mathrm{I} & 0 \end{pmatrix}, \ \mathbf{\Gamma}_7 = \begin{pmatrix} \mathrm{I} & 0 \\ 0 & -\mathrm{I} \end{pmatrix}. \end{aligned} \tag{23}$$

Using these Γ-matrices and $r^2 = x_{\hat{i}}^2 + \sum_{k=4}^7 x_k^2$ we express the asymptotic tachyon condensate in the same form as (16)

$$T = c_0 \mathbf{\Gamma}_i \frac{x^i}{r} = \frac{c_0}{r} \begin{pmatrix} x^7 \mathrm{I} & \sum_{k=1}^5 x^k \Gamma_k + i x^6 \mathrm{I} \\ \sum_{k=1}^5 x^k \Gamma_k - i x^6 \mathrm{I} & -x^7 \mathrm{I} \end{pmatrix}, \tag{24}$$

which is accompanied by a nontrivial gauge field in the same expression as (17). For $i = \hat{i}$ the RHS of (15) is also calculated by using various commutation relations as

$$\frac{c_0}{r^3}(-\epsilon_{\hat{i}\hat{j}\hat{l}} x^{\hat{j}} x_{\hat{k}} \epsilon^{\hat{l}\hat{k}\hat{m}} \mathbf{\Gamma}_{\hat{m}} + \sum_{k=4}^{7} x^k (x_k \mathbf{\Gamma}_{\hat{i}} - x_{\hat{i}} \mathbf{\Gamma}_k)), \tag{25}$$

which again agrees with $\partial_{\hat{i}} T = c_0(\mathbf{\Gamma}_{\hat{i}}/r - x_{\hat{i}}(\mathbf{\Gamma}_{\hat{l}} x^{\hat{l}} + \sum_{k=4}^7 \mathbf{\Gamma}_k x^k)/r^3)$. Similarly for $i = 4, \cdots, 7$ the Eq. (15) is shown to be satisfied by the generalized monopole solution of codimension 7.

Let us return to the generlized monopole solution of codimension 5. From (17) the asymptotic magnetic field strength is given by

$$F_{ij} = \frac{i}{4r^4}((\delta_i^k r^2 - 2x_i x^k)\sigma_{jk} - (\delta_j^k r^2 - 2x_j x^k)\sigma_{ik} - \frac{x^k x^l}{4}[\sigma_{ik}, \sigma_{jl}]) \tag{26}$$

with $\sigma_{ij} = [\Gamma_i, \Gamma_j]$. For this nontrivial gauge field configuration we can evaluate the generalized magnetic charge

$$Q_m = -\alpha \int_{S^4} \mathrm{Tr}(\frac{1}{c_0} T F \wedge F), \tag{27}$$

whose expression is presented in Ref. [6]. Here we have introduced an appropriate normalization α. The integral in (27) is described by $\int \mathrm{Tr}(\frac{1}{c_0} T F^{ij} F^{kl} dS^m \epsilon_{ijklm})$ with $dS^m \epsilon_{ijklm} = dx_i \wedge dx_j \wedge dx_k \wedge dx_l$, where dS^m is further expressed in terms of the unit vector $\hat{r}^m$ as $dS^m = \hat{r}^m dS$. The Chan-Paton indices over which the trace in (27) is taken, are identified with the spinor indices. The generalized magnetic charge is determined by the asymptotic forms of the tachyon and gauge fields. Substituting (16) and (26) into (27) we must take the trace by using $\mathrm{Tr}\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5 = 1$ that is given by the representation (14). There are useful commutation relations

$$[\sigma_{\hat{i}\hat{k}}, \sigma_{\hat{j}\hat{l}}] = 4(-\delta_{\hat{i}\hat{j}}\sigma_{\hat{k}\hat{l}} - \delta_{\hat{k}\hat{l}}\sigma_{\hat{i}\hat{j}} + \delta_{\hat{i}\hat{l}}\sigma_{\hat{k}\hat{j}} + \delta_{\hat{k}\hat{j}}\sigma_{\hat{i}\hat{l}}), \tag{28}$$

which is derived from (20). The other commutation relations are also obtained by

$$\begin{aligned} [\sigma_{\hat{i}4}, \sigma_{\hat{j}\hat{l}}] &= 4(\delta_{\hat{i}\hat{j}}\sigma_{\hat{l}4} - \delta_{\hat{i}\hat{l}}\sigma_{\hat{j}4}),\ [\sigma_{\hat{i}5}, \sigma_{\hat{j}\hat{l}}] = 4(\delta_{\hat{i}\hat{j}}\sigma_{\hat{l}5} - \delta_{\hat{i}\hat{l}}\sigma_{\hat{j}5}), \\ [\sigma_{\hat{i}4}, \sigma_{\hat{j}4}] &= [\sigma_{\hat{i}5}, \sigma_{\hat{j}5}] = -4\sigma_{\hat{i}\hat{j}},\ [\sigma_{\hat{i}4}, \sigma_{\hat{j}5}] = -4\delta_{\hat{i}\hat{j}}\sigma_{45},\ [\sigma_{\hat{i}\hat{j}}, \sigma_{45}] = 0. \end{aligned} \tag{29}$$

They are summarized into a single form

$$[\sigma_{ik}, \sigma_{jl}] = 4(-\delta_{ij}\sigma_{kl} - \delta_{kl}\sigma_{ij} + \delta_{il}\sigma_{kj} + \delta_{kj}\sigma_{il}) \tag{30}$$

with $i = 1, \cdots, 5$, which turn out to be the commutation relations of the $so(5)$ Lie algebra when they are expressed in terms of $Z_{ij} = \sigma_{ij}/4$. This expression in a single form makes it possible to calculate the trace of (27) as

$$Q_m = -\frac{\alpha}{c_0} \int dS \hat{r}^m \epsilon_{mijkl} \mathrm{Tr}(TF^{ij}F^{kl}) = \alpha 4! \Omega_4, \tag{31}$$

where $\Omega_4 = 8\pi^2/3$ is the volume of the unit four-sphere. This result is compared to $\alpha 2! \Omega_2$ with $\Omega_2 = 4\pi$ for the 't Hooft-Polyakov monopole charge. Here we choose the normalization α as $1/4!\Omega_4$ so that the generalized monopole configuration is considered to have unit magnetic charge.

4 Discussion

In order to demonstrate the tachyon condensation accompanied by the nontrivial gauge field configuration we have used the non-BPS D-brane action proposed in Refs. [10, 11]. Taking account of the adjoint representation Higgs mechanism in the tachyon condensation into the generalized five-dimensional monopole configuration for the world-volume gauge theory representing the unstable system of the four non-BPS D9-branes in the type IIA string theory, we have explicitly constructed the effective world-volume gauge theory describing a pair of two BPS D4-branes. We have used a representation of $SO(5)$ Γ-matrices to extract the nontrivial gauge field configuration from the finite-energy requirement and to show that the generalized monopole has unit magnetic charge.

At first sight, however, there seems to be a subtle difference between our explicit demonstration based on the non-BPS DBI action and the general framework based on the higher K-theory group in Ref. [5]. In the latter a stable generalized monopole itself is interpreted as a single BPS D4-brane which is produced from four coincident non-BPS D9-branes as a bound state. On the contrary in our case a pair of two D4-branes are produced as the quantum fluctuations around a generalized monopole solution.

To reconcile with the argument of the higher K-theory group we assume the classical magnetic defect to be identified with a classical BPS D4-brane background. The quantum gauge fluctuations around the static magnetic solution yield a pair of two quantum D4-branes. The magnetic charges of the topological defects may be identified not with the R-R charges of the quantum D-branes but with those of the classical D-brane backgrounds which are classified by the higher K-theory group of the spacetime manifold. The emergences of the lower-dimensional BPS D-branes through the Higgs mechanism in the tachyon condensations

from the non-BPS D9-branes are reminiscent of the appearances of the higher-dimensional fluctuating BPS D-branes in the Matrix theory as the quantum fluctuations around the static and classical BPS D-brane configurations with the nontrivial D-brane charges [17]. In the former the building block is a space-time filling non-BPS D9-brane, whereas in the latter it is a BPS D0-brane.

A more detailed investigation of the interrelations among the adjoint representation Higgs mechanism in the non-BPS D-brane action, the homotopy structure of the classical configuration space and the higher K-theory group is desirable to have a deeper understanding of the features of the classical and quantum BPS D-branes in the dynamics of unstable D-brane systems. It is interesting to demonstrate that our system obtained by the one step construction can be reproduced by the step by step construction and pursue whether there are any differences between the two constructions. Another possible extension would be to apply our prescription to the tachyon condensation in the system of D9-branes and D9-antibranes in the type IIB string theory, where the instanton solution accompanied by the Higgs field and its generalized solutions may play important roles.

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